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A ZENER–STROH CRACK NEAR AN INTERFACE

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Abstract—A micro crack can be initiated by coalescing dislocations piled up along a slip plane. This mechanism was firstly proposed by Zener and later on analyzed by Stroh. The micro crack is thus called Zener–Stroh crack, which is a counterpart of the well-known Griffith crack in linear elastic fracture mechanics. In the present paper, we consider a Zener–Stroh crack initiated near a bi-material interface. Due to image force acted on the dislocations, a Griffith crack mechanism is introduced even where the crack is purely loaded by net dislocations. It is seen that the stress intensity factor, which consists of a Zener–Stroh component and a Griffith component, and the critical crack length are strongly affected by the presence of the interface. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The Zener–Stroh crack, sometimes being called Zener crack or some other names, comes from its own history. It was Zener in 1947 who proposed that dislocations piled up along a slip plane could coalesce into a micro crack at the leading dislocation (see Fig. 1a). In his model, dislocations were stopped by an obstacle where a crack is thus initiated to release high level energy accumulated in the dislocation pileup. Other mechanisms of micro crack initiation via dislocation pileups were observed and shown in Fig. 1b and Fig. 1c. The as initiated micro crack was first analyzed by Stroh (1954) for homogeneous materials (Fig. 1a). After a series of analyses and arguments (Stroh (1955)), Stroh concluded that the initiation of a Zener crack in a solid is possible provided that the energy for the crack initiation is lower than that for other mechanisms, such as dislocation climbing. Analytical work for the configuration shown in Fig. 1c was carried out recently by Cherpanov (1994) for isotropic bi-material and Fan (1994) for general anisotropic linear elastic bi-material by using the so called Stroh formalism (1958). Since the initiated crack is on the interface, complexities, such as crack tip oscillatory behavior, are involved. Nonetheless, all key features of a Zener–Stroh crack are still there.

The energy concept in Zener–Stroh crack and traditional Griffith's crack was generalized by Cottrel and others (see Weertman, 1986). When a crack is loaded by both net dislocations (b_T) and external constant stress traction (σ), the energy stored in the system as a function of crack length is schematically depicted in Fig. 2. It is seen that there are two values of critical crack length due to two mechanisms, namely Zener–Stroh and Griffith mechanisms. A brief explanation is that the stress intensity factor now consists of two components K_{ZS} and K_C.

$$K_{ZS} = \frac{Gb_T}{\sqrt{2\pi a}}, \quad K_G = \sigma\sqrt{\pi a}, \tag{1}$$

due to the Zener–Stroh and Griffith mechanism, respectively. In the equation, a is the crack length, and the critical crack length a_{cr} can be obtained by

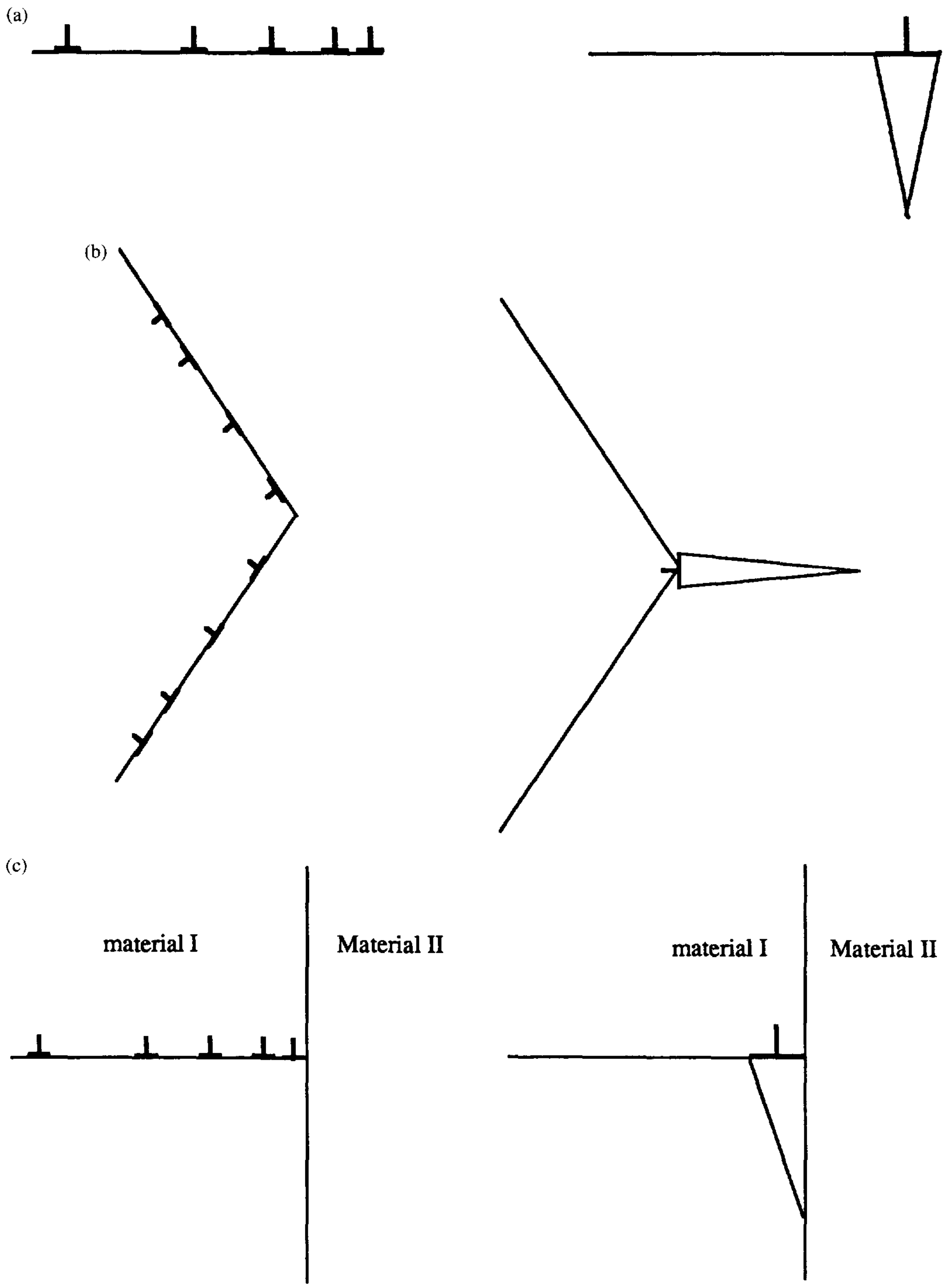


Fig. 1. (a) Zener's mechanism of crack initiation; (b) Cottrel's model of crack initiation; (c) crack initiation at the interface of a bi-material.

$$(K_{ZS}+K_C)^2 = \left(\frac{Gb_T}{\sqrt{2\pi a_{cr}}}+\sigma\sqrt{\pi a_{cr}}\right)^2 = K_C^2. \tag{2}$$

The above equation is a second order equation for the critical crack length and thus has two solutions for a_{cr}. The shorter one a_{cr}^- is under the Zener crack mechanism and is energy wise stable; while the longer one a_{cr}^+ corresponds to the Griffith crack mechanism which is energy wise unstable. Increasing dislocation loading (b_T) pushes the a_{cr}^- value higher; while increasing stress traction loading forces the a_{cr}^+ value lower. It is not difficult to see that the

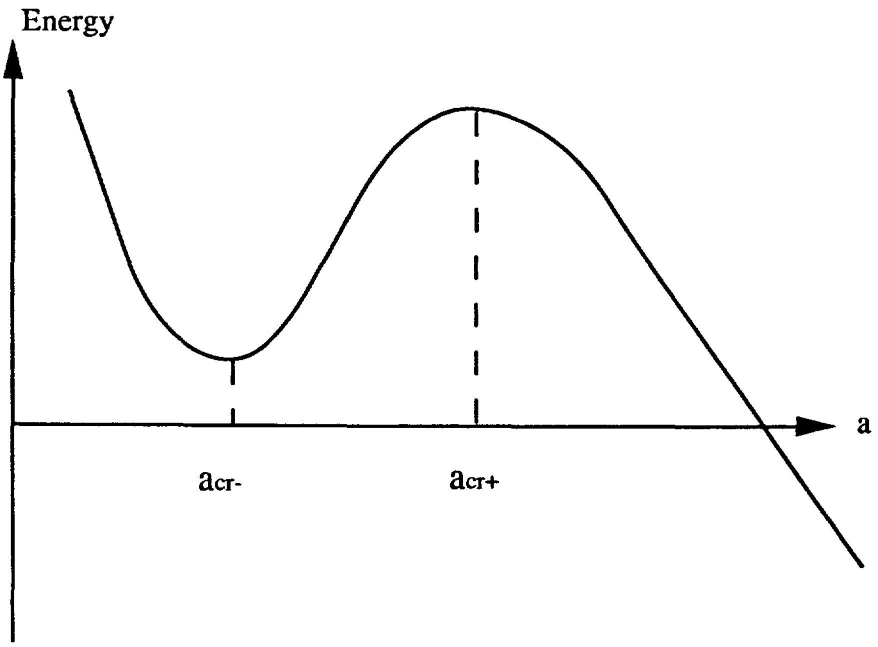


Fig. 2. Critical crack lengths.

Zener–Stroh mechanism controlled critical crack length gives the original size of a micro crack which is initiated from dislocation pileups; while the Griffith mechanism controlled critical crack length predicts the moment of catastrophic failure of the material.

In the present paper, we consider Zener–Stroh cracks near an interface, as shown in Fig. 3a and Fig. 3b. From the solution of a single dislocation near an interface (Dundurs, 1969a), we know that the dislocation is under a stress field of its own image caused by the bi-material interface. By using the equivalency concept between a crack and a pileup of dislocations, a Zener–Stroh crack loaded by net dislocations is also under stress traction on the crack surfaces due to the presence of dislocation images introduced by the interface. The stress intensity factor will be comprised by two parts, namely, the Zener–Stroh component and Griffith component even there is no external traction added on at infinity. As another important parameter for Zener–Stroh crack, the critical crack length will also be affected by the presence of the interface. The size of this critical crack length, although it is still stable in terms of energy, will be enlarged or suppressed depending on combination of the bimaterial moduli.

In order to focus our attention on new issues raised by interface and Zener–Stroh crack, a Mode III crack is analyzed in the following sections where only one bi-material constant is involved in this anti-plane problem. On the other hand, the more important cases, Mode I and II cracks, involve two bi-material constants (Dundurs, 1969b) and crack tip oscillatory behavior. Nonetheless, our analysis and discussion as well as the numerical scheme in the following sections can be applied to Mode I and II without any difficulties. The results for Mode I and II will be presented in a follow-up paper for the sake of completeness of the research on this topic.

2. A ZENER–STROH CRACK PERPENDICULAR TO AN INTERFACE

2.1. *Governing equation*

Since we are going to make a crack equivalent to a pileup of dislocations, we start our formulation with the solution of a single dislocation near an interface. The stress field of a screw dislocation near the interface, which was firstly presented by Head in 1953, was given by Dundurs (1969a) in a very neat form. Refer to Fig. 4 for coordinate and geometric parameters, a single screw dislocation with Bergs vector b_z located at $(t, 0)$, with the branch

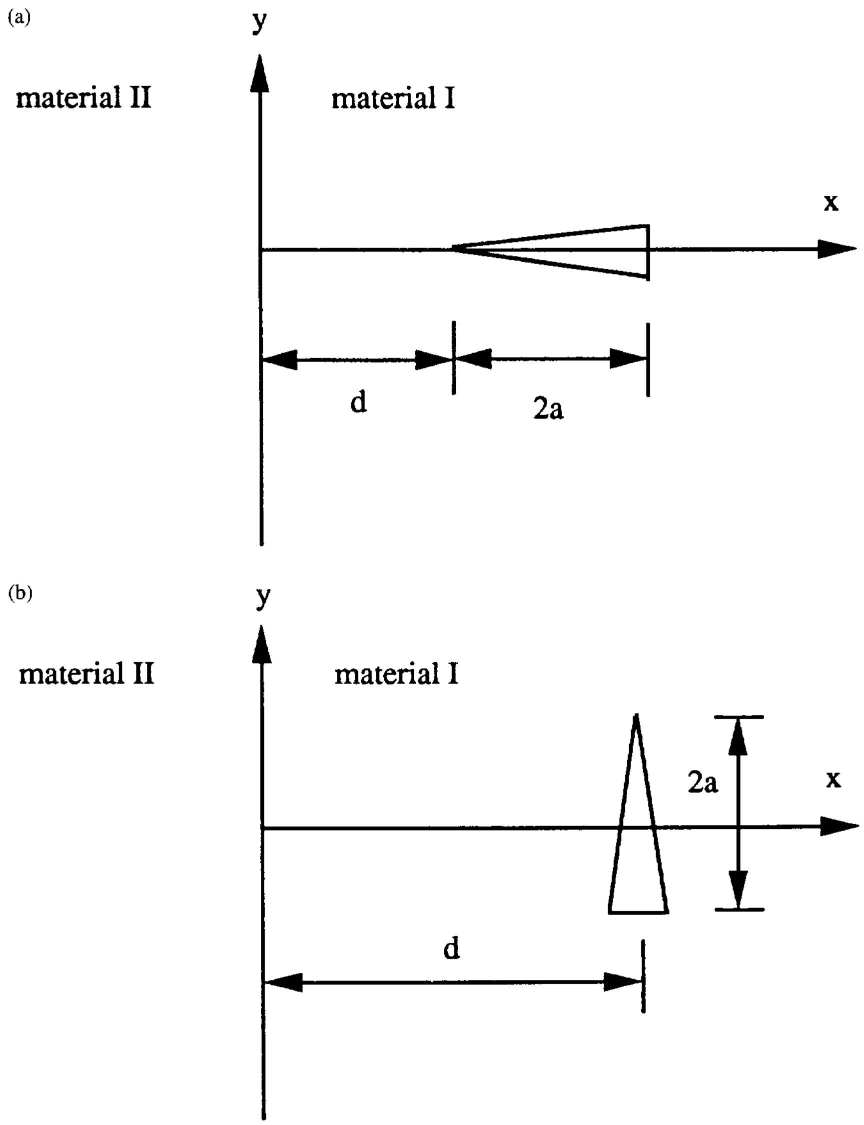


Fig. 3. (a) A Zener–Stroh crack perpendicular to the interface; (b) a Zener–Stroh crack parallel to the interface.

cut line along negative x-axis, the displacement and stress field for any given point (x, y) caused by the dislocation are obtained as

$$u_z^{(1)} = \frac{b_z}{2\pi}(\theta_1 + \gamma\theta_2) \tag{3}$$

$$u_z^{(2)} = \frac{b_z}{2\pi}((1-\gamma)\theta_1 + \gamma\pi) \tag{4}$$

$$\sigma_{zx}^{(1)} = G_1 \frac{\partial u_z^{(1)}}{\partial x} = \frac{G_1 b_z}{2\pi}\left(\frac{\partial \theta_1}{\partial x} + \gamma \frac{\partial \theta_2}{\partial x}\right) \tag{5}$$

$$\sigma_{zy}^{(1)} = G_1 \frac{\partial u_z^1}{\partial y} = \frac{G_1 b_z}{2\pi}\left(\frac{\partial \theta_1}{\partial y} + \gamma \frac{\partial \theta_2}{\partial y}\right) \tag{6}$$

where

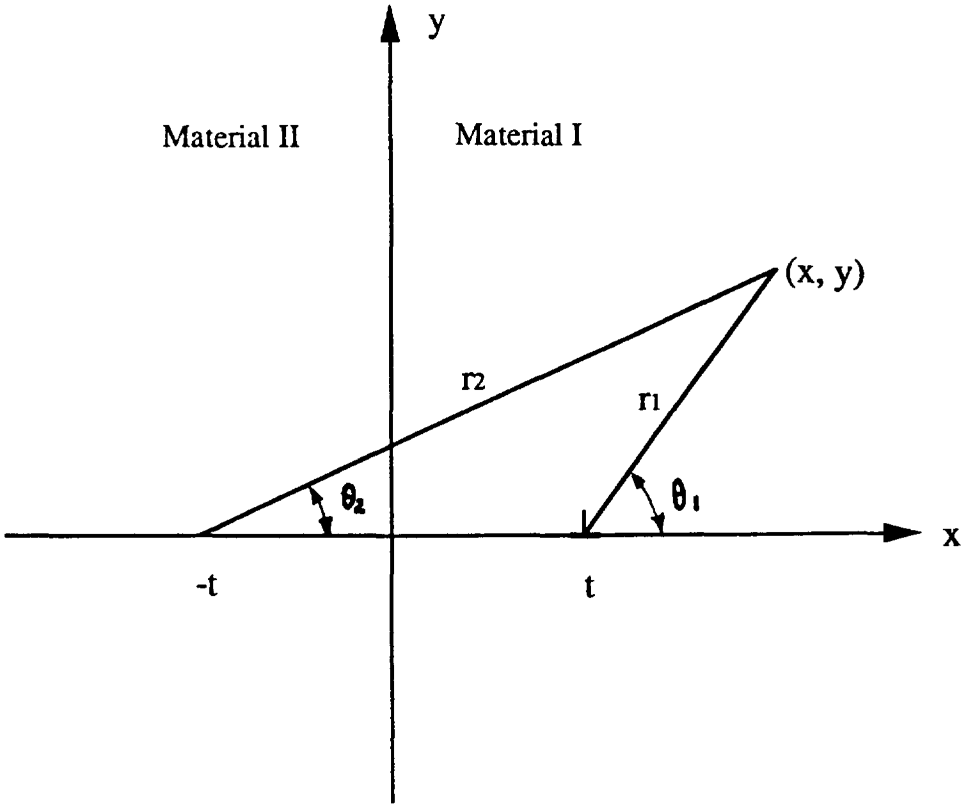


Fig. 4. A dislocation near an interface.

$$\gamma = \frac{G_2 - G_1}{G_2 + G_1}, \quad \tan\theta_1 = \frac{y}{x-t}, \quad \tan\theta_2 = \frac{y}{x+t},$$

G_1 and G_2 are the shear moduli of materials 1 and 2, respectively. Expressing θ_1 and θ_2 in terms of r_1, r_2, x, y and t, the stress $\sigma_z^{(1)}$ is given by

$$\sigma_{zy}^{(1)} = \frac{G_1 b_z}{2\pi}\left(\frac{x-t}{r_1^2} + \gamma\frac{x+t}{r_2^2}\right), \tag{7}$$

when the point (x, y) is on the x-axis, $r_1 = x-t$, $r_2 = x+t$. Applying the solution (7) to a Mode III crack in Fig. 3(a), enforcement of traction free condition on the crack surfaces leads to:

$$\int_d^{d+2a} \frac{G_1 b(t)}{2\pi}\left(\frac{1}{x-t} + \frac{\gamma}{x+t}\right) \mathrm{d}t = 0 \tag{8}$$

where $b(t)$ is the to be determined dislocation density function which satisfies

$$\int_d^{d+2a} b(t)\,\mathrm{d}t = b_T \tag{9}$$

for a Zener–Stroh crack; while its counterpart, the Griffith crack, is solved under the following condition:

$$\int_{d}^{d+2a} b(t)\,\mathrm{d}t = 0. \tag{10}$$

If there is external loading on the crack surfaces, the right hand side in eqn (8) will not be zero.

2.2. *Approximate solution*

To further extend our formulation, a dimensionless system is needed. By introducing

$$X = \frac{x-(d+a)}{a}, \quad T = \frac{t-(d+a)}{a}, \quad B(t) = \frac{b(t)}{a}, \tag{11}$$

eqn (8) can be rewritten as:

$$\int_{-1}^{1} B(T)\left(\frac{1}{X-T} + \frac{\gamma}{X+T+2(d+a)/a}\right)\mathrm{d}T = 0, \tag{12}$$

in which X, $T \in [-1, 1]$. In general, eqn (8) or eqn (12) can only be solved numerically. However, an approximate analytical solution can be obtained when the crack is located relatively far away from the interface, i.e., $d/a \gg 1$. With this condition, eqn (12) can be approximated as:

$$\int_{-1}^{1} B(T)\left(\frac{1}{X-T} + \frac{a}{2(d+a)}\gamma\right)\mathrm{d}T \approx 0. \tag{13}$$

Solutions to eqn (13) can be found in textbooks (e.g. Hirth and Lothe (1982), page 769), which is

$$B(X) = B^{ZS}(X) + B^{G}(X) \tag{14}$$

where the superscripts ZS and G stand for the Zener–Stroh and Griffith parts, respectively. They are

$$B^{ZS}(X) = \frac{b_T}{\pi a(1-X^2)^{1/2}}, \tag{15}$$

$$B^{G}(X) = \frac{Xb_T}{\pi a(1-X^2)^{1/2}}\frac{a\gamma}{2(d+a)}, \tag{16}$$

$$\frac{b_T}{a} = \int_{-1}^{1} B(X)\,\mathrm{d}X = \int_{-1}^{1} B^{ZS}(X)\,\mathrm{d}X. \tag{17}$$

The separation of eqns (15) and (16) is carried out based on the definition of eqns (9) and (10).

2.3. *Stress intensity factors and critical crack length*

The stress intensity factor of the crack can be obtained from the dislocation density function directly via (Weertman, 1992),

$$K = \lim_{X \to 1^-} \left(G\sqrt{\frac{\pi}{2}}\sqrt{a}(1-X)^{1/2}B(X) \right) \tag{18}$$

for the left crack tip, and

$$K = \lim_{X \to -1^+} -\left(G\sqrt{\frac{\pi}{2}}\sqrt{a}(1+X)^{1/2}B(X) \right) \tag{19}$$

for the right crack tip. With the decomposition of eqn (14), the total stress intensity factor consists of two parts, namely, the Zener–Stroh mechanism and the Griffith mechanism. We denote them as

$$K = K_{ZS} + K_G. \tag{20}$$

Since the crack was initiated from a dislocation pileup, the crack tip where the dislocation entered the crack is called blunt tip, while the other tip is called sharp tip. Although there is stress singularity near the blunt tip, the stress at that tip is compressive. As a result the stress intensity factor at the blunt tip is negative. Therefore the crack propagation is always initiated from the sharp tip. Due to this reason, our numerical results just show the stress intensity factors for the sharp crack tip. It is worthwhile to mention that in eqns (15)–(17), $b_T < 0$ corresponds to the sharp tip nearer to the interface than the blunt tip (Fig. 3a), while $b_T > 0$ is the case where the sharp tip is farther from the interface.

When the crack is located far from the interface, the dislocation density function is approximated by eqn (15) and eqn (16), the stress intensity factors ratio is given by

$$\frac{K_G}{K_{ZS}} = \pm \frac{a\gamma}{2(d+a)}, \tag{21}$$

where the positive and negative signs correspond to the left and right crack tips, respectively. This approximate solution will be used to check our numerical scheme in the next section. Furthermore, eqn (21) tells us two important facts, i.e., the Griffith component in total stress intensity factor tends to zero as d increases, and changes its sign according to γ.

The critical crack length is obtained by solving

$$K^2 = (K_{ZS} + K_G)^2 = K_c^2,$$

where for a given net dislocation (Weertman, 1992),

$$K_C = \frac{b_T G_1}{2\sqrt{\pi a_{cr}^{\infty}}},$$

in which a_{cr}^{∞} is the critical crack length when there is no interface interaction.

2.4. *Numerical solution*

To find the dislocation density function $B(X)$ for the general case, a numerical method is employed to solve the integral eqn (12) which is the dimensionless form of (8). Equation (12) is a typical singular integral equation of the first kind:

$$\frac{1}{\pi}\int_{-1}^{1} \frac{B(T)}{X-T}\mathrm{d}T + \int_{-1}^{1} k(X,T)B(T)\,\mathrm{d}T = f(X), \quad -1 < X < 1 \tag{22}$$

where

$$k(X,T) = -\frac{\gamma}{\pi}\frac{B(T)}{X+T+2(d+a)/a}, \quad f(X) \equiv 0$$

for the present case.

To obtain the numerical solution of $B(T)$ in eqn (22), Gaussian integration formula (Erdogan, 1975) is followed. Let

$$B(X) = F(X)(1-X^2)^{-1/2}, \quad -1 < X < 1, \tag{23}$$

where $F(X)$ is a bounded function in $[-1, 1]$. Following Erdogan (1975), the interval $(-1\ 1)$ is divided into n small regions according to

$$T_k = \cos\frac{(2k-1)\pi}{2n}, \quad k = 1,\ldots,n; \quad X_r = \cos\frac{\pi r}{n}, \quad r = 1,\ldots,n-1.$$

The integral eqn (22) and (9) are discreted to the following system of linear algebraic equations in the unknowns $F(T_1),\ldots,F(T_n)$:

$$\sum_{k=1}^{n}\frac{1}{n}F(T_k)\left[\frac{1}{T_k-X_r}+\pi k(X_k,T_k)\right] = f(X_r), \quad r = 1,\ldots,n-1. \tag{24}$$

$$\sum_{k=1}^{n}\frac{\pi}{n}F(T_k) = \frac{b_T}{a}. \tag{25}$$

Equations (24) and (25) offer n linear algebraic equations to solve the n unknowns $F(T_k)$ $(k = 1,\ldots,n)$. Let $\tilde{F}(T)$ be the numerical solution by taking $(b_T/\pi a) = 1$ in (25), then

$$B(T) = \frac{b_T}{\pi a\sqrt{1-T^2}}\tilde{F}(T). \tag{26}$$

By substituting (26) into (18), the total stress intensity factor is obtained as

$$K = \frac{Gb_T}{2\sqrt{\pi a}}\tilde{F}(1), \tag{27}$$

for the case where the sharp crack tip is at $X = 1$.

To find the critical length a_c for every distance d, the following equation is used

$$(K_{ZS}+K_G)^2|_{a=a_c} = K^2|_{a=a_c} = K_C^2. \tag{28}$$

When d goes to infinity, $K_G = 0$, eqns (15), (18) and (20) lead to

$$K^\infty = \frac{Gb_T}{2\sqrt{\pi a}}.$$

Let a_c^∞ be the critical crack length at infinity, noting that the Griffth stress intensity factor at infinity is zero, then

$$K_C = \frac{Gb_T}{2\sqrt{\pi a_c^\infty}}. \tag{29}$$

Substitution of eqns (27) and (29) into (28) yields a simple result as, if the sharp crack tip is at $X = 1$,

$$\frac{a_c}{a_c^\infty} = [\tilde{F}(1)]^2, \tag{30}$$

where $\tilde{F}(1) \approx \tilde{F}(T_1)$ for large n is the numerical solution from (24) and (25) by taking $b_T/\pi a = 1$ and replacing (d/a) in function $k(X, T)$ with (d/a_c). Since

$$\frac{d}{a_c} = \left(\frac{d}{a_c^\infty}\right) \bigg/ \left(\frac{a_c}{a_c^\infty}\right), \tag{31}$$

for every dimensionless distance (d/a_c^∞), eqn (30) can be solved by iteration process to get the critical crack length (a_c/a_c^∞). The above derivations can also be applied to the case where the sharp crack tip is at $X = -1$.

2.5. *Discussions*

The numerical solutions for the variations of K, the total stress intensity factor at the sharp tip, and a_c the critical length, with the distance d at different values of γ, are shown in Figs 5 and 6, for the two cases where the sharp tip is toward and backward the interface, respectively. In the figure, K is normalized by

$$K^\infty = \frac{Gb_T}{2\sqrt{\pi a}}, \tag{32}$$

which is the total stress intensity factor when the distance d is infinity. $\gamma < 0$ is the case where material 2 is "softer" than material 1, while $\gamma > 0$ means material 2 is "harder" than material 1. There are two special cases, i.e., $\gamma = -1$ for free surface $(G_2 = 0)$ and $\gamma = 1$ when material 2 is rigid. One interesting result is that Figs 5(b) and 6(b) indicate there is only one critical crack length even though the stress intensity factor consists of both the Zener–Stroh and Griffith components. In order to check whether another solution of the critical crack length has been missed during the numerical iteration procedure to find a_c, the following argument is considered.

The critical crack length a_c is evaluated through the equation

$$f(a) = K^2 - K_c^2 = 0, \tag{33}$$

where K is the total stress intensity factor and can be written by

$$K = \frac{Gb_T}{2\sqrt{\pi a}} g\left(\gamma, \frac{d}{a}\right), \tag{34}$$

in which $g(\gamma, d/a)$ is the function numerically shown in Figs 5(a) and 6(a). If the equation in (33) has two roots for a_c, there must exist a value of a such that the derivative of $f(a)$ is zero. With the aid of (34), we obtain

$$\frac{\mathrm{d}f(a)}{\mathrm{d}a} = 2K\frac{\partial K}{\partial a} = -K\frac{Gb_T}{2a\sqrt{\pi a}}\left[g\left(\gamma, \frac{d}{a}\right) + \frac{d}{a}g'\left(\gamma, \frac{d}{a}\right)\right], \tag{35}$$

where $g'(\gamma, d/a)$ is the derivative of $g(\gamma, d/a)$ regarding to the variable d/a while γ is kept constant. This derivative is evaluated numerically from the data shown in Figs 5(a) and 6(a). We found that the expression

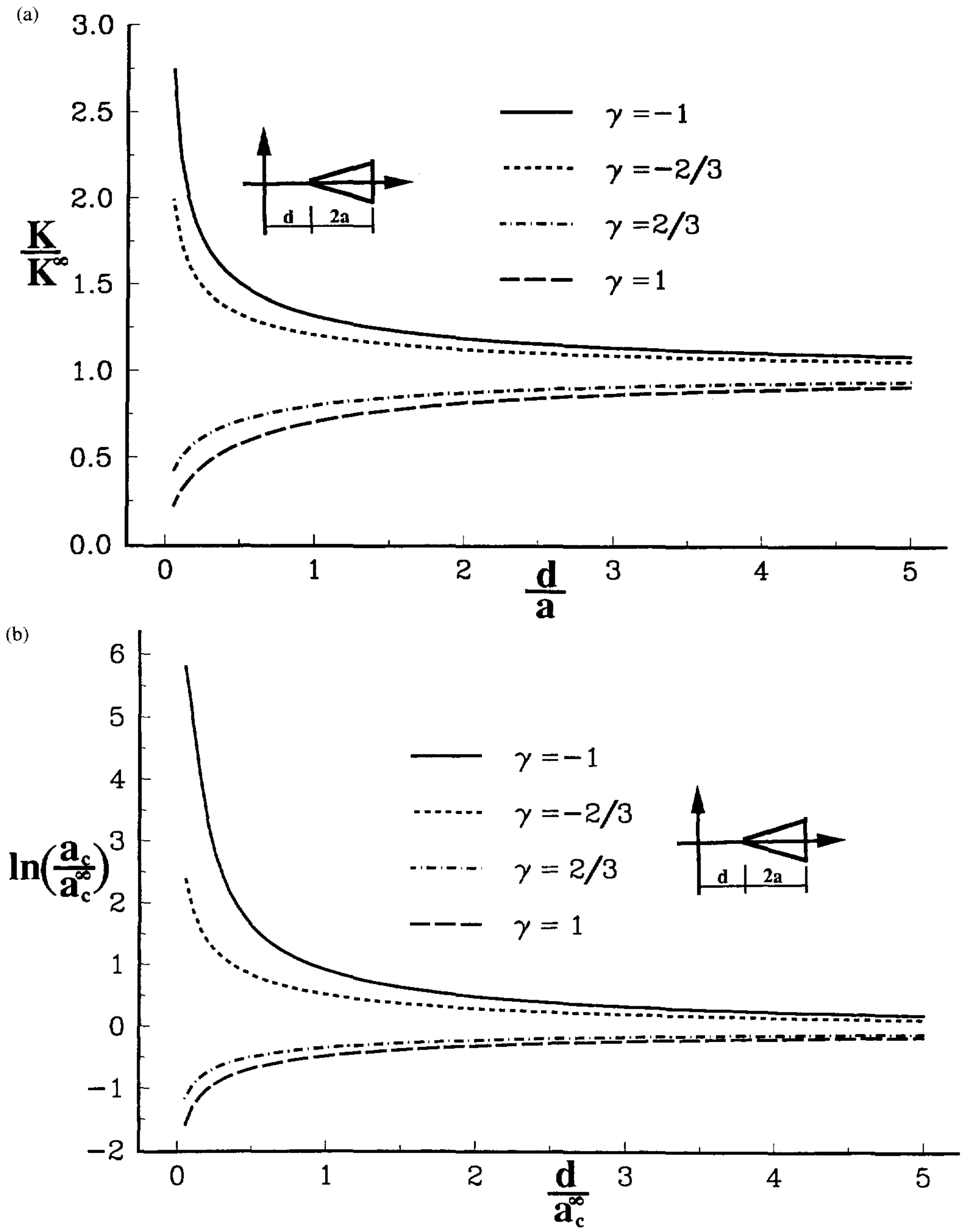
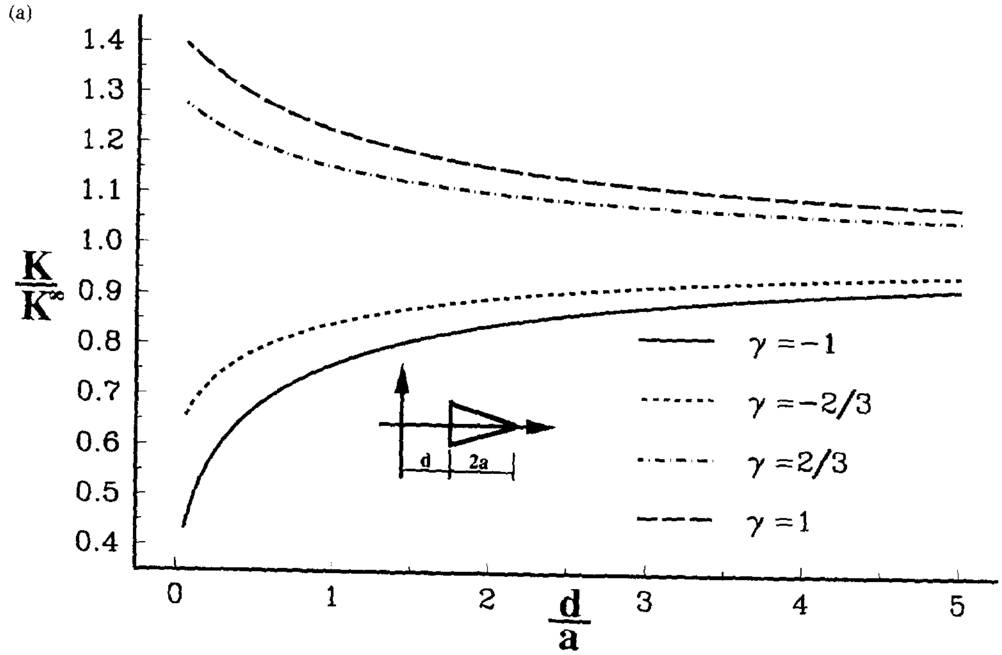


Fig. 5. (a) The total SIF at the sharp crack tip which is towards to the interface; (b) the critical crack length a_c, normalized by its value when d is infinity.

$$g\left(\gamma,\frac{d}{a}\right)+\frac{d}{a}g'\left(\gamma,\frac{d}{a}\right)$$

is always positive. In other words, eqn (35) can not be zero. Therefore, there is only one critical crack length for the current case. The mechanism here is different to Cottrel's energy concept explained in the introduction where the equation for solving a_c is of the order 2 by assuming that the stress applied on the crack is constant. In the current case, the stress acting on the crack due to the interface is a function of d/a, as a result, the equation to evaluate a_c for our case will be no more a second order equation as shown in (2).



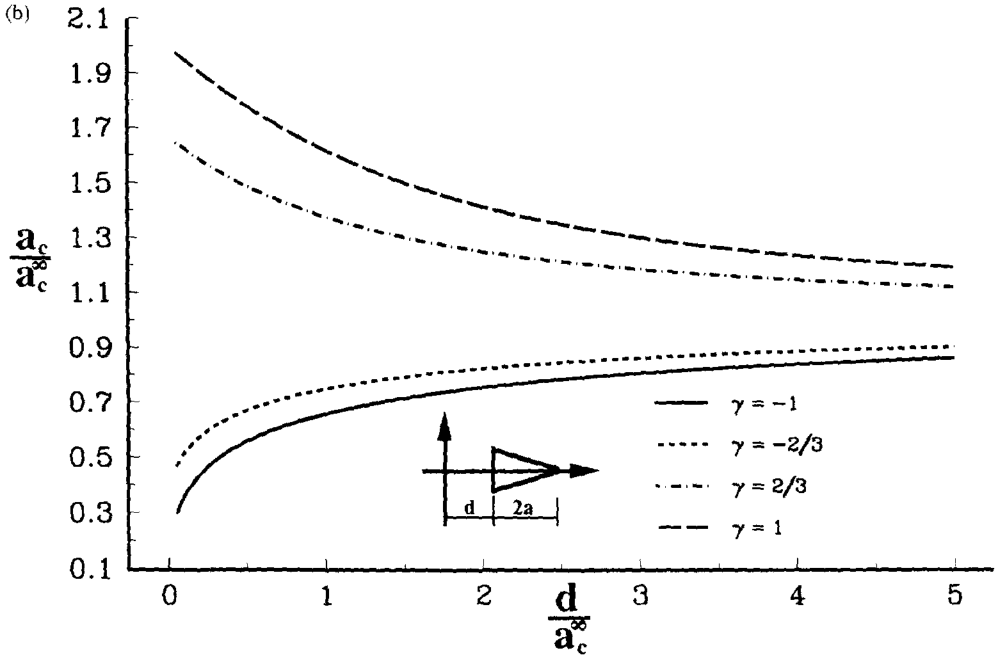


Fig. 6. (a) The total SIF at the sharp crack tip which is backwards to the interface; (b) the critical crack length a_c, normalized by its value when d is infinity.

3. A ZENER–STROH CRACK PARALLEL TO AN INTERFACE

3.1. *Solution*

As the second configuration, we consider a Zener–Stroh crack parallel to an interface as shown in Fig. 3(b). The solution for an arbitrary position of a crack near an interface can be obtained by composing the results in the previous and present sections. Referring to the coordinates shown in Fig. 4, and a single dislocation solution (Dundurs 1969a), we have

$$\int_{-a}^{a} b(t)\left(\frac{1}{y-t}+\frac{\gamma(y-t)}{4d^2+(y-t)^2}\right)\mathrm{d}t = 0. \tag{36}$$

In a dimensionless coordinate,

$$Y = y/a, \quad T = t/a, \quad Y, T \in [-1, 1].$$

Equation (36) is read as:

$$\int_{-1}^{1} B(T)\left(\frac{1}{Y-T} + \frac{\gamma(Y-T)}{4(d/a)^2 + (Y-T)^2}\right) \mathrm{d}T = 0, \tag{37}$$

where the dislocation density function $B(T)$ will be solved under the condition

$$\int_{-1}^{1} B(T)\,\mathrm{d}T = \frac{b_T}{a}.$$

Before we present the general numerical solution for the integral equation, an approximate solution is obtained for $d \gg a$. Equation (37) is rewritten as:

$$\int_{-1}^{1} \frac{B(T)}{Y-T}\mathrm{d}T = -\frac{\gamma}{4(d/a)^2}\int_{-1}^{1} (Y-T)B(T)\,\mathrm{d}T. \tag{38}$$

Noting that $B(Y)$ is a symmetric function of Y, we have

$$\int_{-1}^{1} \frac{B(T)}{Y-T}\mathrm{d}T = -\frac{\gamma b_T Y}{4(d/a)^2}. \tag{39}$$

Then the dislocation density distribution is obtained as

$$B(Y) = \frac{2Y^2-1}{\pi(1-Y^2)^{1/2}}\frac{\gamma b_T}{4\bar{d}^2 a} + \frac{b_T}{\pi(1-Y^2)^{1/2}a} \tag{40}$$

where

$$\frac{b_T}{a} = \int_{-1}^{1} B(T)\,\mathrm{d}T, \quad \bar{d} = \frac{d}{a}.$$

Upon the substitution of eqn (40) into (18), the stress intensity factor for this dislocation distribution is

$$K = \frac{G_1 b_T}{2\sqrt{\pi a}}\left(1 + \frac{\gamma}{4\bar{d}^2}\right). \tag{41}$$

Numerical solutions for the stress intensity factor and critical crack length obtained

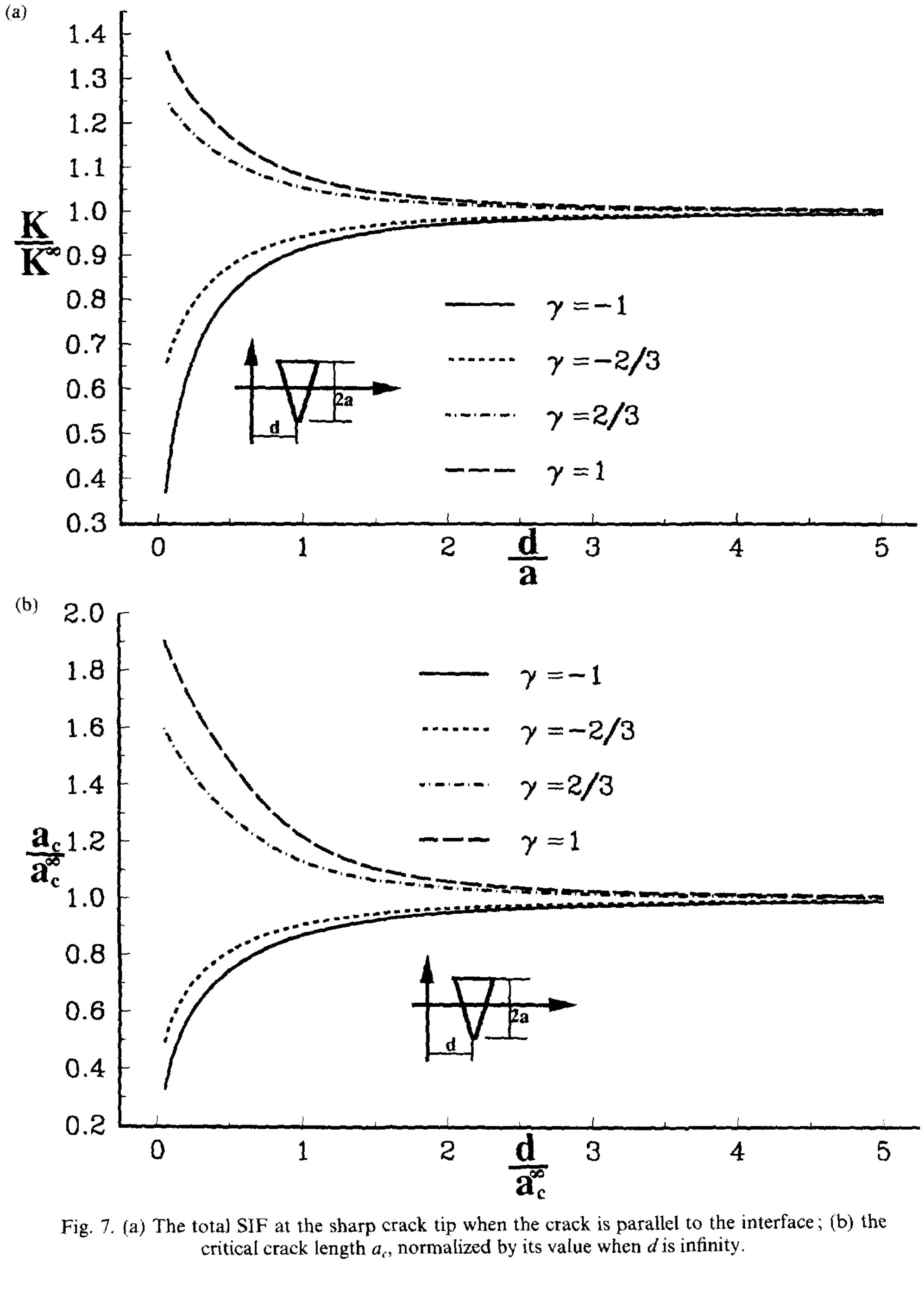


Fig. 7. (a) The total SIF at the sharp crack tip when the crack is parallel to the interface; (b) the critical crack length a_c, normalized by its value when d is infinity.

according to the same procedure as the previous section are shown in Fig. 7(a) and 7(b). The result for $d \gg a$ is checked by eqn (41).

3.2. *Discussion*

It is noticed that an interface crack solution can be reached by setting $d = 0$ in eqn (36) or (37). The dislocation density reads as

$$B_{int}(Y) = \frac{b_T}{\pi a(1-Y^2)^{1/2}}. \tag{42}$$

However, the stress intensity factor presented in Fig. 7a does not converge to the stress intensity factor of an interface crack as d tends to zero. This fact has been noticed by other researchers in the field of interface fracture mechanics (Hutchinson *et al.*, 1987). As d/a goes to zero, applicable region of the crack tip K-field in material I is approaching to zero too due to the fact that the crack is now very close to the interface. On the other hand, an interface crack tip field is formed outside the above mentioned crack tip field for a homogeneous material. These two crack tip asymptotic expansions can be connected by path independent integrals. For in-plane problem, due to the intrinsic mixed mode of the interface crack, the connection between "inner" (homogeneous material K-field) and "outer" (interface crack tip field) field is more complex (Hutchinson *et al.*, 1987). This argument can also be applied to the case formulated in Section 2 where the crack is perpendicular to the interface.

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